Underwater Passive Target Tracking Using Unscented Kalman Filter

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ABSTRACT: In surveillance sonar passive tracking for bearing measurements has been done so far by using kalman filter and its modifications. In this paper, target is expected to be moving typically at constant speed in underwater scenario. Unscented Kalman Filter has been utilised to track underwater submarine target / moving ship. Monte Carlo Simulation in PC environment is used to prove the effectiveness of UKF. Cases taken up are Ownship/Target S or L manoeuvring. Methods applied include for target tracking, mathematical modelling, and bearings only techniques. Simulation results have been obtained and comparison has been made based on typical scenarios. Different configurations of underwater scenarios can be taken up for further research.

Keywords: **Extended kalman filter, unscented kalman filter, Bearings only tracking, Target motion analysis, S-manoeuvring**

I. INTRODUCTION

In underwater scenario, the sonar plays a prominent role. SONAR (“Sound Navigation and Ranging”) systems have many similarities to radar and electro- optical systems. The process of sonar is based on the propagation of sound waves between a target and a receiver. The main purpose of sonar is the detection or classification (estimation of position, velocity, and identity) of submerged, floating, or buried objects. The two most common types of sonar systems are passive and active. Passive sonar includes a receiver but no transmitter. The signal to be detected is then the sound emitted by the target. In an active sonar system, waves propagate from a transmitter to a target and back to a receiver. Determination of the position of target is termed target motion analysis (TMA). TMA is done by using the measurements required from the target. And these measurements are Doppler frequency and bearings. As these measurements are combined with noise. In underwater, the noise in the measurements is very high, turning rate of the platforms is low and speed of the platforms is also low when compared with the missiles in air. In order to eliminate this noise we require filters, Wiener filter and kalman filter. These are the linear systems. Rudolf E. Kalman was the first developer of Kalman filter in 1960.The Kalman Filter is a special kind of observer that provides optimal filtering of process and measurement noises if the covariances of these noises are known. But in real world no system is linear. So we are going to use nonlinear system. These are two types and these nonlinear systems are EKF (Extended kalman filter) and UKF (Unscented kalman filter) although EKF is used for nonlinear systems but in EKF the state distribution is approximated by GRV (Gaussian random variable). But it is correct up to 1st order. So it is not used for higher nonlinearities. So UKF is used. In the UKF the state distribution is approximated by the sigma points are transformed through the nonlinear function. Here the UKF is used to estimate the state from the noisy measurements and these noisy measurements are "Doppler frequency and bearing from the passive sonar".

Assumptions

Following are the assumptions made in the simulator.

(1) At start, Ownship is at the origin.

(2) Target is moving at constant velocity and

(3) We follow True North convention

A) Own ship manoeuvring

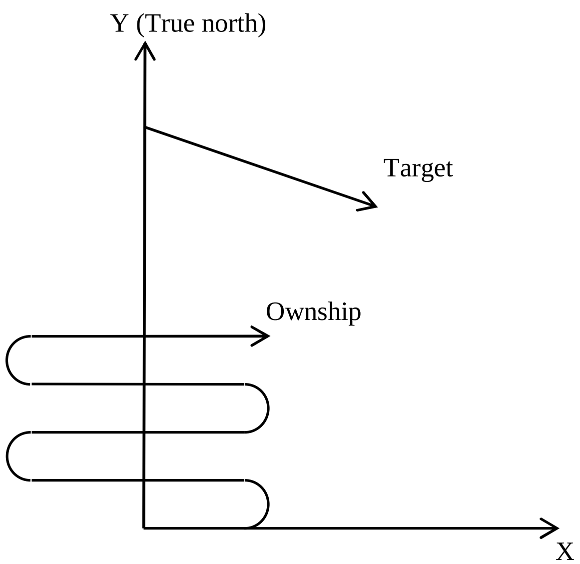


Fig: Ownship in S-manoeuvre

Ownship follows either S manoeuvre or manoeuvre recommendation. For every second change in x and y component of Ownship position is found and added to the previous x,y components of O/S position.

For ts = 1 sec

$$dxo = vo*\sin(ocr)*ts \quad (1)$$

$$dyo = vo*\cos(ocr)*ts \quad (2)$$

where

- dxo is change in x-component of own ship position in 1 sec.
- dyo is change in y-component of own ship position in 1 sec.
- vo is Ownship velocity
- ocr is Ownship course (True North)
- (xo,yo) is Ownship position

xo = (xo+dxo) and yo = (yo+dyo). In typical situation, own ship motion is often restricted to straight line segments and each segment is termed as "leg". It is well known that to obtain an observable process, for the purpose of estimating the complete target state, an Ownship manoeuvre is required. Generally in Navy S-manoeuvre for Ownship is adapted.

B) Target manoeuvring

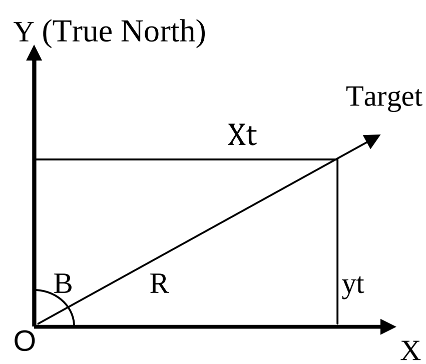


Fig. 2.T-Target

Where

B: Bearing, R: Range

From input range and bearing initial position of target is known.

For ts = 1 sec

xt = range*sin(bearing) (3)

yt = range*cos(bearing) (4)

(xt , yt) is target position with respect to Ownship as the origin.

For every 1 sec, changes in xt and yt are calculated and added to previous target position.

dxt = vt*sin(tcr) *ts (5)

dyt = vt*cos(tcr) *ts (6)

xt = xt+dxt and yt = yt+dyt

Where

dxt is change in x component of target position in one second

dyt is change in y component of target position in one second

vt is target velocity

tcr is target course with respect to true north

Range and Bearing are generated by the following formulae:

Where

True range= $\sqrt{(xt - xo)*(xt - xo) + (yt - yo)*(yt - yo)}$ (7)

true bearing = $Tan^{-1}\dfrac{(xt - xo)}{(yt - yo)}$ (8)

II. Target tracking and mathematical modeling

Passive target tracking is the determination of the trajectory of a target solely from measurements of signals originating from the target. Modern underwater vehicles use multi sensors to track multi targets. For example, modern ships use Towed Array (TA) along with Hull Mounted Array (HMA). TA is used to obtain measurements from target at long ranges when compared to that of HMA.

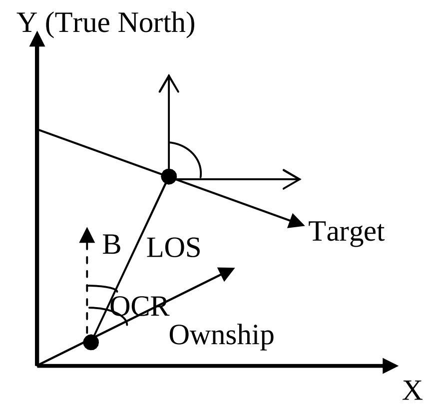


Fig 4.Target and observer encounter

Where

B: Bearing

LOS : Line Of Sight

OCR : Own submarine Course

TCR : Target Course

State and measurement equations:

The target is assumed to be moving with constant velocity as shown in the fig1. And is defined to have the state vector. Let the target state vector be

$$x_s(k) = [\dot{x}(k)\ \dot{y}(k)\ R_x(k)\ R_y(k)]^T \quad (9)$$

Where $\dot{x}(k)$ and $\dot{y}(k)$ are target velocity components and, R_X (k) and R_y (k) are range components respectively. The target state dynamic equation is given

$$x_s(k+1) = \varphi x_s(k) + b(k+1) + \Gamma\omega(k) \qquad (10)$$

Where φ and b are transition matrix and deterministic vector respectively. The transition matrix is given by

$$\varphi = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ t & 0 & 1 & 0 \\ 0 & t & 0 & 1 \end{bmatrix}$$

Where t is sample time

$$b(k+1) = \begin{bmatrix} 0 & 0 & -\{x_o(k+1) - x_o(k)\} & -\{y_o(k+1) - y_o(k)\} \end{bmatrix} \qquad (11)$$

And $\Gamma = \begin{bmatrix} t & 0 \\ 0 & t \\ \frac{t^2}{2} & 0 \\ 0 & \frac{t^2}{2} \end{bmatrix}$

Where are observer position components.

The plant noise, $\omega(k)$ is assumed to be zero mean white Gaussian with $E\left[\Gamma(k)w(k)w^1(j)\Gamma^1(j)\right] = Q\delta_{ij}$

$$Q = \begin{bmatrix} t_s^2 & 0 & \frac{t_s^3}{2} & 0 \\ 0 & t_s^2 & 0 & \frac{t_s^3}{2} \\ \frac{t_s^3}{2} & 0 & \frac{t_s^4}{8} & 0 \\ 0 & \frac{t_s^3}{2} & 0 & \frac{t_s^4}{8} \end{bmatrix}$$

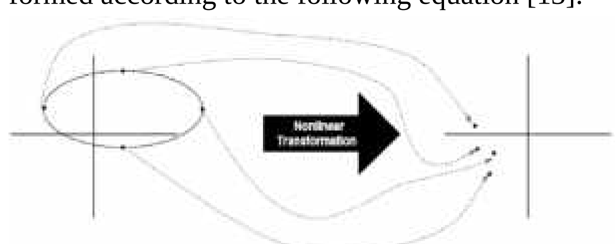
True North convention is followed for all angles to reduce mathematical complexity and for easy implementation. The bearing measurement, Bm is modeled as

$$B_m(k+1) = \tan^{-1}\left(\frac{R_x(k+1)}{R_y(k+1)}\right) + \varsigma(k) \qquad (12)$$

Where, $\varsigma(k)$ is error in the measurement and this error is assumed to be zero mean Gaussian with variance$\pm^{2}$.

III. UNSCENTED KALMAN FILTER ALGORITHM

The traditional Kalman filter is optimal when the model is linear. Unfortunately, many of the state estimation problems like tracking of the target using bearings-only information are nonlinear, thereby limiting the practical usefulness of the Kalman filter and EKF. The unscented transformation is coupled with certain parts of the classical Kalman filter called as UKF. Unscented transformation is designed to propagate information in the form of mean vector and covariance matrix through a nonlinear process, is explored for underwater applications. The unscented transformation is coupled with certain parts of the classical Kalman filter. Unscented transformation is a method for calculating the statistics of a random variable which undergoes a nonlinear transformation. Consider a random variable x (dimension L_1) propagating through a nonlinear function $y = o(x)$. Assume x has mean $\bar{x}$ and covariance P_x. To calculate the statistics of y, a matrix χ of $2L_1 + 1$ sigma vectors χ_i (with corresponding weights W_i), is formed according to the following equation [13]:



$$\chi_0 = \bar{x}$$
$$\chi_i = \bar{x} + \left(\sqrt{(L_1+\lambda)P_x}\right)_i \quad i = 1,...,L_1$$
$$\chi_i = \bar{x} - \left(\sqrt{(L_1+\lambda)P_x}\right)_{i-L_1} \quad i = L_1+1,...,2L_1$$
$$W_0^{(m)} = \lambda/(L_1+\lambda)$$
$$W_0^{(c)} = \lambda/(L_1+\lambda) + \left(1 - g^2 + \xi\right)$$
$$W_i^{(m)} = W_i^{(c)} = 1/2(L_1+\lambda) \quad i = 1,...,2L_1$$

(13)

Where $\lambda = g^2(L_1+\kappa) - L_1$ is a scaling parameter g determines the spread of the sigma points around $\bar{x}$ and is usually set to a small positive value (e.g., 1e-3), κ is a secondary scaling parameter which is usually set to 0 and ¶ is used to incorporate prior knowledge of the distribution of x (for Gaussian distributions, $\xi = 2$ is optimal). $\left(\sqrt{(L_1+\lambda)P_x}\right)_i$ is the i^{th} row of the matrix square root. $W_0^{(m)}, W_0^{(c)}$, $W^{(m)}$ and $W^{(c)}$ are the weights of initialized target state vector, covariance matrix of initialized target state vector, target state sigma point vector and covariance matrix of target state sigma point vector respectively. These sigma vectors are propagated through the nonlinear function

$$y_i = o(\chi_i) \quad i = 1,...,2L_1$$

(14)

The mean and covariance are approximated using a weighted sample mean and covariance of the posterior sigma points [13]. UKF is a straightfor-ward extension of the unscented transformation to the recursive estimation. In UKF, the state random variable is redefined as the concatenation of the original state and noise variables. The unscented transformation sigma point selection scheme is applied to this new augmented state random variable to calculate the corresponding sigma matrix. The standard UKF implementation consists of the following steps:

(a) Calculation of the $(2n+1)$ state vectors with sigma points starting from the initial conditions, where n is dimension of target state vector

$$X(k) = \left[X_S(k) \quad X_S(k) + \sqrt{(n+\lambda)P(k)} \quad X_S(k) - \sqrt{(n+\lambda)P(k)}\right]$$

(15)

(b) Transformation of these sigma points through the process model using eqn.

(c) The prediction of the state estimate at time $k+1$ with measurements up to time k is given as

$$X_S(k+1,k) = \sum_{i=0}^{2n} W_i^{(m)} \; X_S(i,(k+1,k))$$

(16)

(d) As the process noise is additive and independent, the predicted covariance is given as

$$P(k+1,k) = \sum_{i=0}^{2n} W_i^{(c)} [X_S(i,(k+1,k)) - X_S(k+1,k)] \times [X_S(i,(k+1,k)) - X_S(k+1,k)]^T + Q(k)$$

(17)

(e) Updation of the sigma points with the predicted mean and covariance: The updated sigma points are given as

$$X(k+1,k) = \begin{pmatrix} X_S(k+1,k) \; X_S(k+1,k) + \\ \sqrt{(n+\lambda)P(k+1,k)} \; X_S(k+1,k) - \\ \sqrt{(n+\lambda)P(k+1,k)} \end{pmatrix}$$

(18)

(f) Transformation of each of the predicted points through measurement model eqn.

(g) Prediction of measurement given as

$$\hat{z}(k+1,k) = \sum_{i=0}^{2n} W_i^{(m)} . Y(k+1,k)$$

(18)

Where

$$Y(k+1,k) = h(X_S(k+1,k))$$

(19)

(h) Since the measurement noise is also additive and independent, the innovation covariance is given as

$$P_{yy} = \sum_{i=0}^{2n} W_i^{(c)} \left[Y(i,(k+1,k)) - \hat{z}(k+1,k) \right] \times \left[Y(i,(k+1,k)) - \hat{z}(k+1,k) \right]^T + \sigma_B^2(k) \tag{20}$$

(i) The cross covariance is given as

$$P_{xy} = \sum_{i=0}^{2n} W_i^{(c)} \left[X_S(i,(k+1,k)) - X_S(k+1,k) \right] \times \left[Y(i,(k+1,k)) - \hat{z}(k+1,k) \right]^T \tag{21}$$

(j) Kalman gain is calculated as

$$G(k+1) = P_{xy} . P_{yy}^{-1} \tag{22}$$

(k) The estimated state is given as

$$X(k+1,k+1) = X(k+1,k) + G(k+1)\left(\hat{z}(k+1,k+1) - \hat{z}(k+1,k)\right) \tag{23}$$

Where

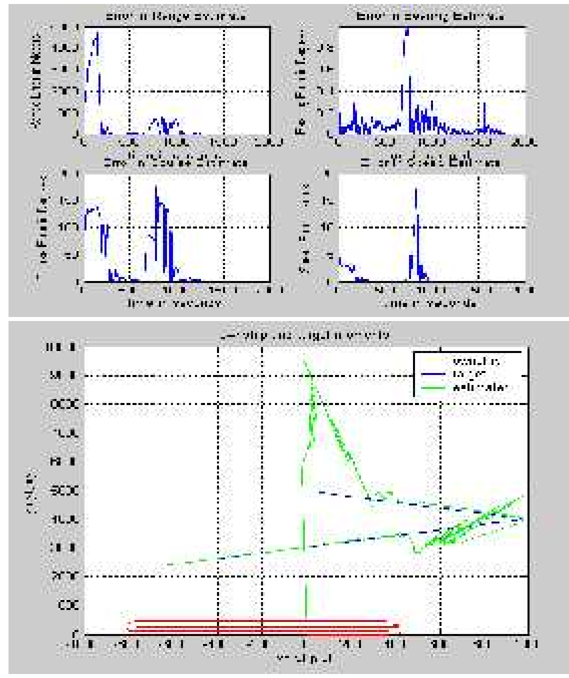
$z(k+1)$ is measurement vector.

(l) Estimated error covariance is given as

$$P(k+1,k+1) = P(k+1,k) - G(k+1) P_{yy} G^T(k+1) \tag{24}$$

IV. SIMULATION

All raw bearings and elevation measurements are corrupted by additive zero mean Gaussian noise with a r.m.s level of 0.33 degree. Corresponding to a tactical scenario in which the target is at the initial range of 5000 meters at initial bearing and 5 degrees respectively. The target is assumed to be moving at a constant course of 200 degrees. Observer is assumed to be stationary or manoeuvring. The results have been ensemble averaged over several Monte Carlo runs. The errors in estimates are plotted in Fig.2. It is observed that this required accuracy is obtained from 1000 seconds onwards and so this algorithm seems to be very much useful for underwater passive target tracking when Observer is stationary or manoeuvring, for a moving target.



V. CONCLUSION

The paper deals with simulation of the motion of the target and determining the initial target parameters namely bearing and frequency. These parameters were then corrupted with noise to get the noisy measurements. BOT [5] is a right method to obtain target motion parameters without using Ownship manoeuvre. This method can be easily adopted for underwater passive target tracking application. In this paper, an approach using a UKF (which is useful for nonlinear applications) is proposed to estimate target motion parameters without using Ownship manoeuvre in passive target tracking. Monte-Carlo simulation was carried out in the scenarios. Therefore we may conclude that UKF is robust algorithm.

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